A New Radiation Boundary Condition for FDTD based on Self-Teleportation of fields

Rudy Diaz and Igor Scherbatko

Arizona State University, Tempe, AZ, 85287-5706, USA

Abstract — **In [1] a technique for injecting perfect plane waves into finite regions of space in FDTD was reported. The essence of the technique, called Field Teleportation, is to invoke the principle of equivalent sources using FDTD's discrete definition of the curl to copy any field propagating in one FDTD domain to a finite region of another domain. In this paper we apply this technique of Field Teleportation to the original domain itself to create a boundary across which any outward traveling FDTD field produces an exact negative copy of itself. Repeated application of this radiation boundary and termination of the domain with a moderate absorbing boundary condition readily yields reflection coefficients of the order of –80 dB up to grazing incidence onto the boundary.**

I. INTRODUCTION

The advent of Berenger's PML [2] started a new era in FDTD modeling. Since then many improvements have been made to the concept of the Absorbing Boundary Condition [3-6] to the point that any practitioner willing to spend the computational resources can create extremely quiet FDTD zones for performing numerical electromagnetics experiments. However, implementing a PML is not a trivial undertaking, especially for the material intersections and terminations. For these reasons it would be desirable to create a new Radiation Condition that is broadband in frequency and angle, and at the same time easy to program. In this paper it is shown that such a Radiation Boundary Condition (RBC) can be implemented using the discrete version of Schelkunoff's equivalent currents. This approach was used in [1] to teleport perfect plane waves into finite regions of FDTD with no leakage. Here the RBC effect is obtained by teleporting outgoing fields back into the FDTD domain with a negative sign, thus canceling outward traveling waves.

In Section II of this paper we show how discrete equivalent currents are included in the regular FDTD update procedure and thus allow the teleportation of fields. In Section III the Field Teleportation scheme is implemented within the source grid to create the RBC. After comparing this RBC with the standard PML in free space, we demonstrate its excellent performance as a termination of a domain with an inhomogeneous boundary between a lossless and a lossy dielectric.

II. TELEPORTATION OF FIELDS

If the constitutive properties of the materials modeled inside the FDTD space are assumed from the outset to be dispersive, then both the DC electric conductivity and the analogous fictitious DC magnetic conductivity can be absorbed into the permittivity and permeability operators, $\varepsilon(t)$, $\mu(t)$. For instance, the simplest dispersive material composed of a constant permittivity and a DC conductivity has as its time domain operator permittivity:

$$\boldsymbol{\varepsilon}(t) = \varepsilon_r + \left\lfloor \int dt \frac{\sigma}{\varepsilon_0} \right. \tag{1}$$

where the half-bracket term signifies that the integral is an operator that acts on the functions to the right of it. In this paper we use the convention that bold-faced variables contain (dispersive) time domain operators. Equation (1) is the time domain version of the usual frequency domain expression: $\varepsilon(\omega)=\varepsilon_r+\sigma/(j\omega\varepsilon_0)$; and it may be readily verified that it leads to the more traditional form in equation 2 for the case of a time invariant conductivity.

$$\frac{\partial \bar{\mathbf{D}}}{\partial t} = \frac{\partial\left(\boldsymbol{\varepsilon}(t)\bar{E}\right)}{\partial t} = \frac{\partial\left(\varepsilon_r \bar{E}\right)}{\partial t} + \sigma\bar{E} = \frac{\partial \bar{D}}{\partial t} + \bar{J}_e \tag{2}$$

Clearly all currents induced in a material are taken into account by such dispersive constitutive operators. Therefore Maxwell's first Curl equation, in the absence of impressed currents, can always be written in the form:

$$\nabla \times \bar{H} = \frac{\partial \bar{\mathbf{D}}}{\partial t} \tag{3}$$

Now, the update equation procedure of FDTD can be understood as application of equation 3, to derive the updated Displacement current from the curl of H, followed by derivation of the updated Electric field from the auxiliary differential equation connecting E to D[6]. This is a standard viewpoint adopted when FDTD is applied to dispersive materials. Note that as would be expected from its time derivative nature, the Displacement current exists at the half-integer timesteps, in synchrony with the magnetic field.

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For the case of the simplest material of equation 1, the update equation given in equation (4) is an adequate connection between D and E for most conductivities of interest; and we use it as an example because of its simplicity. (Of course if the relaxation time in the conducting material is short compared to dt, then the so-called exponential time-stepping expression can be used. It will be clear from the derivation that follows that the procedure described below applies equally well to any arbitrary dispersive material).

$$E^{n+1} = \left(\frac{1-\frac{\sigma dt}{2\varepsilon_r\varepsilon_0}}{1+\frac{\sigma dt}{2\varepsilon_r\varepsilon_0}}\right)E^n + \frac{dt}{\varepsilon_r\varepsilon_0}\left(\frac{1}{1+\frac{\sigma dt}{2\varepsilon_r\varepsilon_0}}\right)\left(\frac{\partial D}{\partial t}\right)^{n+\frac{1}{2}} \quad (4)$$

In equation (4), the first term on the right describes the decay of the Electric field from last timestep as a result of conductivity-driven diffusion. The second term on the right describes an incremental E field, call it ΔE^{n+1}, that results from the source term (in this case the Displacement current).

Now consider the case when an impressed current is included in equation (3). Clearly, to perform the update, the equation must be written as:

$$\frac{\partial \mathbf{\bar{D}}}{\partial t} = \nabla \times \bar{H} - \bar{J}_e \quad (5)$$

So that an impressed current enters into the update equation (4) through the Displacement current. In other words, in the presence of an impressed current in this material medium, there is an additional incremental E field given by:

$$\Delta E^{n+1} = \frac{dt}{\varepsilon_r\varepsilon_0}\left(\frac{1}{1+\frac{\sigma dt}{2\varepsilon_r\varepsilon_0}}\right)\left(-J_e\right)^{n+\frac{1}{2}} \quad (6)$$

The importance of equation (6) is that it shows that all impressed currents must be processed through the dispersive constitutive properties to be properly applied inside the FDTD update cycle. Now, we know that Schelkunoff's equivalent currents allow us to recreate a total field outside a closed volume bounded by a surface S by following the prescription

$$\bar{K}_e = \hat{n} \times \bar{H}_{tot}, \bar{K}_m = -\hat{n} \times \bar{E}_{tot} \quad (7)$$

where n is the outward surface normal to S. Clearly then, any total field existing inside one FDTD domain at a boundary surface (taken for simplicity to be a plane normal to one of the principal axes) can be turned into a set of equivalent surface currents lying on an identical boundary surface in another FDTD domain in such a way that the currents radiate exactly the same field in the second domain as exists in the first. In other words, the fields have been teleported from one domain to another. When these currents are correctly implemented in FDTD, they behave exactly as Schelkunoff's currents, creating the correct field on the "right" side of S and zero field on the left side. There is no leakage, as shown in reference [1].

Recognizing that in the discrete space of FDTD, $J_e = K_e/ds$, with ds being the size of the space cell, equation (6) tells us that the incremental fields are given by:

$$\begin{aligned} \Delta E^{n+1} &= \frac{dt}{\varepsilon_r\varepsilon_0}\left(\frac{1}{1+\frac{\sigma_e dt}{2\varepsilon_r\varepsilon_0}}\right)\left(-\frac{\hat{n}\times\bar{H}}{ds}\right)^{n+\frac{1}{2}} \\ \Delta H^{n+\frac{1}{2}} &= \frac{dt}{\mu_r\mu_0}\left(\frac{1}{1+\frac{\sigma_m dt}{2\mu_r\mu_0}}\right)\left(\frac{\hat{n}\times\bar{E}}{ds}\right)^{n} \end{aligned} \quad (8)$$

In a typical FDTD computer program the implementation would be as follows. (For the sake of simplicity only the case of a 2DTM space is given). The four possible teleportation walls of interest would be a wall that teleports to the left along the x-axis, one that teleports to the right along the x axis, one that does so downwards along the y axis, or upwards along the y axis, located at i_{left}, i_{right}, j_{bottom}, j_{top} (where I is the integer index for cells along x and j for cells along y). Along those walls, the following quantities are stored in a buffer matrix immediately after the E-field loop:

For all desired i:

$$\begin{aligned} E_{temp}(i, j_{top}) &= A_{et} \cdot H_z(i, j_{top}) \\ E_{temp}(i, j_{bottom}) &= -A_{eb} \cdot H_z(i, j_{bottom}) \end{aligned} \quad (9)$$

where

$$\begin{aligned} A_{et} &= \frac{dt}{\varepsilon_0\varepsilon_r(i, j_{top}+1)dx(1+Q_{et})}, \quad Q_{et} = \frac{\sigma(i, j_{top}+1)dt}{2\varepsilon_0\varepsilon_r(i, j_{top}+1)} \\ A_{eb} &= \frac{dt}{\varepsilon_0\varepsilon_r(i, j_{bottom})dx(1+Q_{eb})}, \quad Q_{eb} = \frac{\sigma(i, j_{bottom})dt}{2\varepsilon_0\varepsilon_r(i, j_{bottom})} \end{aligned} \quad (10)$$

and for j:

$$\begin{aligned} E_{temp}(i_{left}, j) &= A_{el} \cdot H_z(i_{left}, j) \\ E_{temp}(i_{right}, j) &= -A_{er} \cdot H_z(i_{right}, j) \end{aligned} \quad (11)$$

where

$$\begin{aligned} A_{el} &= \frac{dt}{\varepsilon_0\varepsilon_r(i_{left}, j)dy(1+Q_{el})}, \quad Q_{el} = \frac{\sigma(i_{left}, j)dt}{2\varepsilon_0\varepsilon_r(i_{left}, j)} \\ A_{er} &= \frac{dt}{\varepsilon_0\varepsilon_r(i_{right}+1, j)dx(1+Q_{er})}, \quad Q_{er} = \frac{\sigma(i_{right}+1, j)dt}{2\varepsilon_0\varepsilon_r(i_{right}+1, j)} \end{aligned} \quad (12)$$

Similar equations are derived for the H-field. To teleport the field into an identical FDTD space, these buffered values are used as sources as follows:

Along i:

$$\begin{aligned} E_x(i, j_{top}+1) &= E_x(i, j_{top}+1) + E_{temp}(i, j_{top}) \\ E_x(i, j_{bottom}) &= E_x(i, j_{bottom}) + E_{temp}(i, j_{bottom}) \end{aligned} \quad (13)$$

Along j:

$$\begin{aligned} E_y(i_{right}+1, j) &= E_y(i_{right}+1, j) + E_{temp}(i_{right}, j) \\ E_y(i_{left}, j) &= E_y(i_{left}, j) + E_{temp}(i_{left}, j) \end{aligned} \quad (14)$$

A similar procedure is followed with the H-field.

Note that these equations are exactly like the ordinary update equations of FDTD and therefore they work for all time varying fields that can be supported by the FDTD grid. Since the teleported fields are exact copies of the original fields, it is natural to ask what would happen if the teleported fields were injected back into the source domain but with a sign change. This would lead to exact cancellation of the outgoing waves in the domain, the perfect absorbing boundary condition.

III. Self-Teleportation of Fields

It turns out that it is impossible to teleport the fields back onto the source walls because the teleportation recipe gets caught in a feedback loop. However, it is possible to teleport the copied fields one cell beyond where they were collected. To compensate for this one cell shift, the fields are stored one step in time in the past. Therefore the cancellation is not perfect, but it is very good (typically of the order of –20dB, time domain average, per wall). These RBC walls can be cascaded one behind the other by leaving just one cell of separation (again to prevent feedback). The FDTD domain behind the Radiation Boundary Condition can be terminated with any simple one-sided termination.

In this paper, a numerical comparison of the RBC and the unsplit PML is presented. Two schemes were chosen to assess the RBC's quality in terms of evanescent waves and traveling waves incident on the terminal boundary over a wide range of angles. No special attempts have been made to optimize either RBC or PML in testing configuration. The first comparison between RBC and PML has been done for free space modeled as an "anechoic chamber". A square 2D *X-Y* domain 700 by 700 cells in area was alternately terminated with a 10 cell PML or with an RBC field termination layer consisting of a simple one-sided termination and 3 RBC walls. The unsplit PML scheme employs a cubical law of conductivity [4]. A z-directed electric field is injected exactly in the center of the area having a time dependence of the form: $pulse = \sin^3(2\pi n/80)$, $0 \le n \le 80$ and $pulse =$ 0, for n>80.

The total energy within the domain (normalized to its maximum value) as function of the time step n, for both boundary conditions is shown in Figure1. In general, the RBC absorbs outgoing waves much better than PML, giving –65 dB attenuation after a second echo reflection (-35 dB for the PML). Note that the RBC scheme consumes less time and memory. A 6000 time steps run took 330 seconds for the RBC scheme and 583 for the PML (and used 12 Mb versus 20·Mb for the PML).

Near-field RBC absorption has also been studied in the numerical experiment schematically shown in Figure 2. A point source generates continuous wave $\sin(2\pi n/80)$ for 1400 time steps that is enough to propagate a signal in the measurement area without any reflections from the front and back walls (if the source is located at x=700, y=350). The signal distribution (magnitude of the principal Fourier component at the excitation frequency) over the measurement area is shown in Figure 3 for the case when the source is at the center of the domain in *y* and for a second case where it is moved closer to one sidewall.

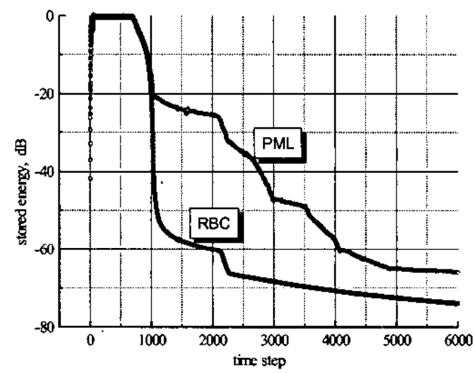


Fig.1. Stored energy versus time step for 10-cells RBC and PML schemes for the anechoic chamber modeling.

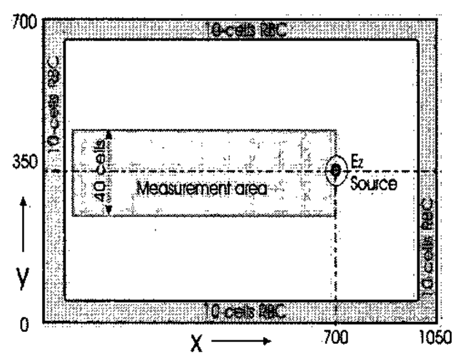
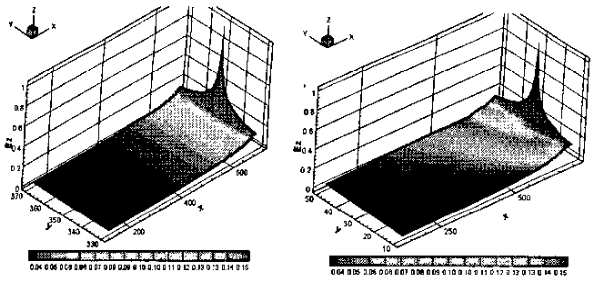


Fig. 2. Scheme of the numerical experiment (initial source position is shown).

It is seen that that when the source is located close enough to the absorption layer (for example at x = 700, y = 30), the reflection from the wall changes the field distribution inside the area (as shown for PML in Fig. 3b). Figure 4 repeats the experiment with the RBC. The perturbation in the case of the RBC is approximately one order of magnitude down compared to the PML.

A more challenging problem for an ABC is that of terminating an inhomogeneous domain, for instance the problem of a source in free space above a lossy dielectric half-space.



a) b)

Fig.3. (a) Magnitude of the unperturbed Ez field (source at x=700,y=350), (b) Magnitude of the Ez field in PML vicinity (source at x=700,y=30), PML layer is 10-cells wide (y=0-10).

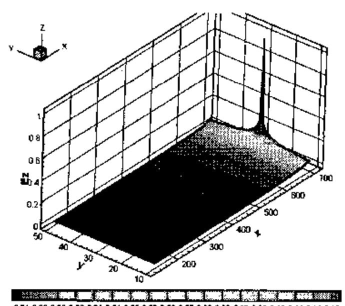


Fig.4. Magnitude of the Ez field in RBC vicinity (source at x=700,y=30).

The ABC is required to cut transparently through both media. In the PML, since the electric and magnetic conductivities differ in both media there is a "matching" problem at the boundary. However, for the RBC, since the attenuation of the outgoing waves is created by exact FDTD copies of the incident waves, all matching occurs automatically. Figure 5, shows a snapshot in time of the problem domain. The field difference just above the RBC, between the RBC-truncated domain and the full 600x600 domain is plotted against an "effective" angle of incidence (obtained by drawing a straight line from the source to the sampled point). The RBC again affords –80 dB attenuation at nearly all angles of incidence and it suffers no discontinuity at the air/lossy-dielectric boundary.

IV. CONCLUSION

A new Radiation Boundary Condition that works for arbitrary time varying fields has been proposed and demonstrated. It is trivial to program into conventional FDTD and it is more angle and material insensitive than the PML.

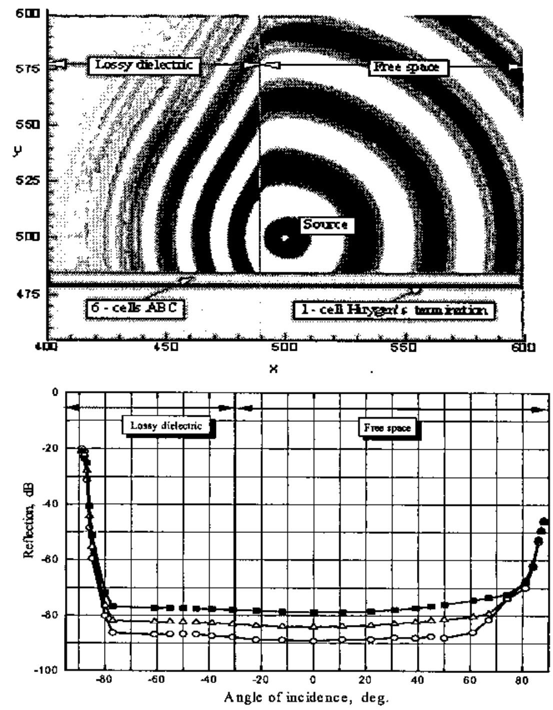


Fig. 5. Snapshot in time of the numerical experiment in an inhomogeneous space and the reflection coefficient as a function of angle of incidence. o:ε_r=4-j0.6, Δ: ε_r=4-j0.75, □: ε_r=4-j1.0

REFERENCES

[1] M. E. Watts, R. E, Díaz, "Perfect Plane-Wave Injection into a finite FDTD domain through Teleportation of Fields", submitted for publication to *Electromagnetics*, 2002.

[2] Berenger J.P., "A perfectly matched layer for the absorption of electromagnetic waves", *J. Comp.Phys.*, vol. 114, 1994, pp. 185-200.

[3] S.D. Gedney, "An anisotropic perfectly matched layer-absorbing medium for the truncation of FDTD lattices", *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 1630-1639, Dec 1996.

[4] D.M. Sullivan, Electromagnetic Simulation Using the FDTD Method. IEEE Press Series on RF and Microwave Technology, 2000.

[5] S.C.Winton and C.M. Rappaport, "Specifying PML conductivities by considering numerical reflection dependencies", *IEEE Trans. Antennas Propagat.*, vol. 48, No. 7, July 2000, pp. 1055-1063.

[6] A. Taflove, *Advances in Computational Electrodynamics: The finite-difference Time-Domain Method*, Artech House,1998.